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Sub-Poissonian three-level lasing with an m-photon coherent pump

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Abstract
We theoretically consider a three-level laser with an incoherent one-way pump from a ground state into an intermediate excited state and a coherent drive into the upper laser state. The possibilities of inversionless lasing are studied. It is demonstrated that, in the case of a strong coupling with the drive laser (when its generation depends essentially on the three-level medium), the quantum properties of laser radiation can be improved by using m-quantum coherent excitation or by applying a sub-Poissonian coherent pump.

Keywords: inversionless lasing, sub-Poissonian light, photodetector, coherent state

1. Introduction

Three-level lasing is of interest because, first, it contains some intrinsic mechanism that ensures sub-Poissonian lasing [1] automatically without some additional efforts to distinguish it from two-level lasers [2], for example, where we need to foresee a regular pump. Second, the system is the simplest available that is able to ensure so called inversionless gain [3]. We are going to discuss both these aspects together.

A laser using a Λ-configuration of levels and statistical properties of its radiation were considered in [1], and it was demonstrated that there is reduction of the shot noise, but the efficiency was not very high. The strongest effect shown is the reduction by halves. It is then natural to search for a simple physical system which is able to overcome this result, and one of the extremely attractive candidates is the three-level laser due to its ability to produce sub-Poissonian lasing automatically.

In this paper, two possible systems will be discussed. First we will propose the coherent pump which is realized by sub-Poissonian light. Obviously we can expect here some additional reduction of the laser emission noise, because, in the system as whole, in principle, the number of noise sources decreases. The same can be expected in the case when we apply the m-photon pump, because, in this case, even the Poissonian light is perceived to be sub-Poissonian.

2. Semi-classical approach

2.1. A three-level laser model

The active medium of the laser consists of three-level atoms, which are schematically shown in figure 1(a). The laser levels are the uppermost (a) and the ground (b), and the coherent pump is in resonance with the adjacent transition ($c-a$) (Λ-configuration). To ensure lasing we must predict some incoherent one-way pump ($b \to c$) with a rate γ_b. And our last additional requirement is for the laser transition ($a-b$) spontaneous emission occurring with a rate χ.

It needs to be stressed that in this configuration there is a conversion of the small frequency of the pump into the higher one of lasing. At the same time our formulae obtained for this model will be perfectly suitable for the configuration represented in figure 1(b) and thereby describe a situation with a reduction of frequency.

To implement a one-way pump ($b \to c$) in a real situation we must take into account some additional level (d) (see figure 2) and some additional pump incoherent mechanisms ensuring processes ($b \to d$), ($d \to b$) and ($d \to c$). Further, in our formal equations just the model is considered.

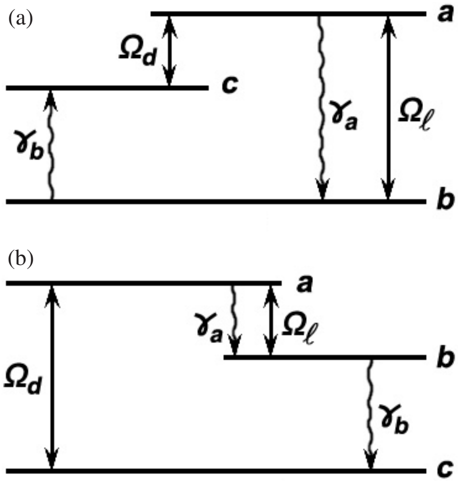

Figure 1. Atomic configurations for the three-level laser.

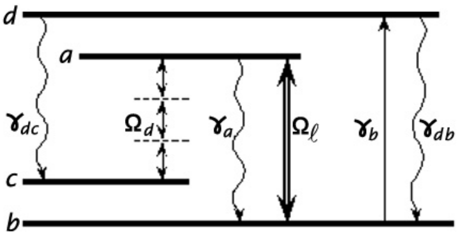

Figure 2. The three-level configuration of atoms with an additional level (d) to implement a one-way incoherent pump.

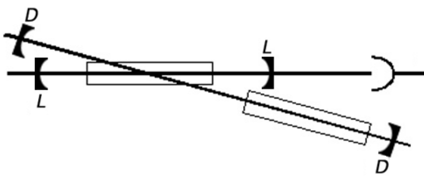

Figure 3. Experimental setup for studying the quantum properties of the emission of the three-level laser.

Under our theoretical description the following conceptual experiment is carried out (figure 3). There are two lasers: exciting D and excited L. The first can be Poissonian or sub-Poissonian depending on our needs. The second is our three-level laser. The excitation of the medium of the second laser is carried out by the intracavity field of the first one (the scheme with common intracavity space).

Let us write a ground equation for the semi-classical theory in the form:

$$\dot{\hat{\sigma}} = -\imath[H_{\mathrm{I}}, \hat{\sigma}] + \hat{R}_{\mathrm{at}}\hat{\sigma}. \tag{1}$$

Here $\hat{\sigma}$ is the four-level atom density matrix (figure 2), the operator $\hat{R}_{\mathrm{at}}$ ensures all the incoherent processes, the interaction Hamiltonian H_{I} in the interaction picture is

$$H_{\mathrm{I}} = -\Omega_{\mathrm{l}}|a\rangle\langle b| - \Omega_{\mathrm{d}}|a\rangle\langle c| + \text{ h.c.} \tag{2}$$

Here Ω_{l} and Ω_{d} are the complex Rabi frequencies for the laser and pump channels, respectively:

$$\Omega_{\mathrm{l}} = -\imath g_{\mathrm{l}}\alpha_{\mathrm{l}}, \qquad \Omega_{\mathrm{d}} = -\imath g_{\mathrm{d}}\alpha_{\mathrm{d}}^{m}, \qquad m = 1, 2, 3, \ldots \tag{3}$$

where α_{l} is the c-number complex amplitudes of the laser fields and α_{d} that of the pump (or drive) field; $g_{\mathrm{l,d}}$ are respective coupling constants. As is seen, the opportunity for m-quantum interaction on the transition $(a-c)$ is taken into account within the framework of the model Hamiltonian (2). For simplicity we choose all the frequency detunings in the system to be equal to zero.

Rewriting the equation in the terms of the atomic matrix elements, we have the following:

$$\dot{\sigma}_{bb} = -\gamma_b\sigma_{bb} + \gamma_a\sigma_{aa} + \gamma_{db}\sigma_{dd} - \imath\Omega_{\mathrm{l}}\sigma_{ba} + \imath\Omega_{\mathrm{l}}^{*}\sigma_{ab} \tag{4}$$

$$\dot{\sigma}_{cc} = \gamma_{dc}\sigma_{dd} - \imath\Omega_{\mathrm{d}}\sigma_{ca} + \imath\Omega_{\mathrm{d}}^{*}\sigma_{ac} \tag{5}$$

$$\dot{\sigma}_{dd} = \gamma_b\sigma_{bb} - \gamma_d\sigma_{dd}, \qquad \gamma_d = \gamma_{dc} + \gamma_{db} \tag{6}$$

$$\sigma_{aa} + \sigma_{bb} + \sigma_{cc} + \sigma_{dd} = 1 \tag{7}$$

and

$$\dot{\sigma}_{ca} = -\tfrac{1}{2}\gamma_a\sigma_{ca} + \imath\Omega_{\mathrm{d}}^{*}(\sigma_{aa} - \sigma_{cc}) - \imath\Omega_{\mathrm{l}}^{*}\sigma_{cb} \tag{8}$$

$$\dot{\sigma}_{cb} = -\tfrac{1}{2}\gamma_b\sigma_{cb} + \imath\Omega_{\mathrm{d}}^{*}\sigma_{ab} - \imath\Omega_{\mathrm{l}}\sigma_{ca} \tag{9}$$

$$\dot{\sigma}_{ba} = -\tfrac{1}{2}(\gamma_a + \gamma_b)\sigma_{ba} + \imath\Omega_{\mathrm{l}}^{*}(\sigma_{aa} - \sigma_{bb}) - \imath\Omega_{\mathrm{d}}^{*}\sigma_{bc}. \tag{10}$$

Within the framework of the semi-classical theory only stationary solutions are of interest to us and that is why we can set all the derivatives over t to be equal to zero. As a result the system of differential equations is converted into a system of algebraic ones. Then, with help of equations (8)–(10) we can express all the non-zero polarizations via the population differences $n_{ik} = \sigma_{ii} - \sigma_{kk}$ in the following form:

$$\sigma_{ab} = -\frac{\imath\Omega_{\mathrm{l}}}{\gamma_{ab}}\frac{(1 + I - (1+y)I_{\mathrm{d}})n_{ab} + (1+y)I_{\mathrm{d}}n_{cb}}{1 + I + I_{\mathrm{d}}} \tag{11}$$

$$\sigma_{ac} = -\frac{\imath\Omega_{\mathrm{d}}}{\gamma_{ab}}\frac{-In_{ab} + (1+y)(1+I_{\mathrm{d}})n_{ac}}{1 + I + I_{\mathrm{d}}} \tag{12}$$

$$\sigma_{cb} = \frac{2\Omega_{\mathrm{l}}\Omega_{\mathrm{d}}^{*}}{\gamma_b\gamma_{ab}}\frac{n_{ab} + (1+y)n_{ac}}{1 + I + I_{\mathrm{d}}}. \tag{13}$$

Here

$$\gamma_{ab} = \tfrac{1}{2}(\gamma_a + \gamma_b), \qquad y = \frac{\gamma_b}{\gamma_a}, \tag{14}$$

and I and I_{d} are dimensionless intensities of the laser and drive fields which are expressed via the respective Rabi frequencies in the form:

$$I = \frac{4|\Omega_{\mathrm{l}}|^2}{\gamma_a\gamma_b}, \qquad I_{\mathrm{d}} = \frac{2|\Omega_{\mathrm{d}}|^2}{\gamma_b\gamma_{ab}}. \tag{15}$$

Using equations (4)–(7) and substituting there (11)–(13), we can get a system of equations for populations:

$$\sigma_{aa}(1+x)\left(1 + I_{\mathrm{d}} + I\frac{y}{1+y}\right) - \sigma_{bb}y\left(1 + I_{\mathrm{d}} + I\frac{x}{1+y}\right) = 0 \tag{16}$$

$$\sigma_{aa}\left(\frac{1}{y} + 2\right)I_{\mathrm{d}} + \sigma_{bb} \times \left[\frac{1+I}{(1+y)(1+x)} + \left(1 + z - \frac{1}{1+x}\right)I_{\mathrm{d}}\right] = I_{\mathrm{d}} \tag{17}$$

where

$$x = \frac{\gamma_{db}}{\gamma_{dc}}, \qquad z = \frac{\gamma_b}{\gamma_{db} + \gamma_{dc}}. \tag{18}$$

2.2. Coherence of the laser transition at the threshold of generation

Now we have all the possibilities to write the required formulae for the stationary populations and polarizations in the general case. However, they are too cumbersome to discuss here; two limits are more important for us. First, to specify the mechanisms of gain, we need to know what happens at a threshold of generation (at $I \ll 1$). Second, the quantum properties of the laser emission are usually the most expressive under a laser saturation condition, i.e. it is the area $I \gg 1$ which is of interest to us. These two cases will be discussed in this section.

Keeping in (11) the common coefficient Ω_l and putting $I = 0$ in all the other places of the formula, the polarization of the laser transition on the threshold of generation can be written in the form:

$$\sigma_{ab} = -\frac{g_l \alpha_l}{\gamma_{ab}} \frac{(1 - I_d - yI_d)n_{ab} + (1+y)I_d n_{cb}}{1 + I_d}. \tag{19}$$

One can see that a gain in the laser transition depends on two factors: on the traditional laser inversion n_{ab}, and on the Raman inversion n_{cb}. The inversions can be written in the explicit form on the basis of stationary solutions for populations on the threshold of generation ($I \ll 1$):

$$\sigma_{aa} = \frac{y(1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]} \tag{20}$$

$$\sigma_{bb} = \frac{(1+x)(1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]} \tag{21}$$

$$\sigma_{cc} = \frac{1 + (1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]} \tag{22}$$

$$\sigma_{dd} = \frac{z(1+x)(1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]}. \tag{23}$$

From here the actual inversions read:

$$n_{ab} = \frac{(y - 1 - x)(1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]} \tag{24}$$

$$n_{cb} = \frac{1 - x(1+y)I_d}{1 + I_d[(1+y)^2 + (1+x)(1+y)(1+z)]}. \tag{25}$$

To conclude relative to gain we can write the laser equation for slow laser amplitude α_l, which is given formally by the following:

$$\dot{\alpha}_l = -\frac{\kappa_l}{2}\alpha_l - Ng_l\sigma_{ab}. \tag{26}$$

Here N is the number of three-level atoms which participate in lasing. For a gain we require $\mathrm{Re}(\sigma_{ab}/\alpha_l) < 0$. It is possible to achieve it in different ways.

For example, putting $x(1 + y)I_d = 1$, we have the possibility of eliminating the processes connected with the Raman inversion, because according to (25) then $n_{cb} = 0$. Then in the case of the three-level medium a gain can take place for both the situations with positive and negative inversion on laser transition. It is not difficult to understand that traditional laser amplification with the positive inversion $n_{ab} > 0$ is achieved under conditions

$$x(1+y)I_d = 1, \qquad y > 1 + x, \qquad x > 1. \tag{27}$$

At the same time the inversionless amplification with $n_{ab} < 0$ takes place when

$$x(1+y)I_d = 1, \qquad y < 1 + x, \qquad x < 1. \tag{28}$$

2.3. Coherency of the laser transition in the saturation regime

For our statistical study it is useful to know solutions in the regime of the laser saturation that is under $I \gg 1$, I_d. Keeping the highest terms we can get the following formulae:

$$\sigma_{aa} = x(1+y)\frac{I_d}{I} \ll 1, \qquad \sigma_{bb} = (1+x)(1+y)\frac{I_d}{I} \ll 1, \qquad \sigma_{cc} \approx 1 \tag{29}$$

$$\sigma_{ab} = -\frac{2g_l\alpha_l}{\gamma_a}\frac{I_d}{I^2}, \qquad \sigma_{ac} = -\frac{2g_d\alpha_d^m}{\gamma_a}\frac{1}{I}, \qquad \sigma_{cb} = -\frac{4g_lg_d\alpha_l\alpha_d^{*m}}{\gamma_a\gamma_b}\frac{1}{I}. \tag{30}$$

Equation (26) provides us with the condition of the stationary generation:

$$n_l = (\sqrt{n_d})^{m+1}\frac{\tilde{\kappa}_d}{\kappa_l}\frac{1}{1+y}. \tag{31}$$

Here the magnitude

$$\tilde{\kappa}_d = -2Ng_d\,\mathrm{Re}\left(\frac{\sigma_{ac}}{\alpha_d}\right) = \frac{\gamma_b(\sqrt{n_d})^{m-1}Ng_d^2}{n_lg_l^2} \tag{32}$$

has a physical sense of speed of the m-photon absorption of driving light in the three-level medium in the transition (a–c).

3. Quantum approach

3.1. The kinetic equation for the density matrix of the laser field

In contrast to the consideration in the previous section we now assume both the laser and the drive fields are quantized and will try to construct a kinetic equation for the field density matrix. The physical system, consisting of two lasers and represented schematically in figure 3, can be discussed theoretically as a two-mode laser with a two-component active medium and a complicated cavity. Let the matrix ρ describe a behaviour of both the modes. Putting the high-Q cavity relative to both modes we can write formally the following kinetic equation:

$$\dot{\rho} = (\hat{S}_d - \hat{R}_d)\rho + (\hat{S}_l - \hat{R}_l)\rho. \tag{33}$$

Here operators $\hat{R}_d$ and $\hat{R}_l$ ensure damping of the quantum field oscillators with rates κ_d and κ_l, respectively, and as is known in the Glauber diagonal representation, they read:

$$\hat{R}_i = \kappa_i\frac{\partial}{\partial u_i}u_i, \qquad i = \mathrm{d}, \mathrm{l}. \tag{34}$$

In contrast to the semi-classical approach, here $\alpha_i = \sqrt{u_i}\exp(\imath\varphi_i)$ are not simply c-numbers but the eigen-numbers of the respective operators of annihilation of photons:

$$a_i|\alpha_i\rangle = \alpha_i|\alpha_i\rangle, \qquad [a_i, a_i^\dagger] = 1. \tag{35}$$

The explicit form of the operator $\hat{S}_\mathrm{d}$ depends on the concrete model of the driving laser. If we, for example, wish to use for excitation of the three-level laser a two-level Poissonian laser, then the explicit form of the operator $\hat{S}_\mathrm{d}$ can be found in work [4]. At the same time for the sub-Poissonian pump we can find the respective expression in work [2].

We must calculate now the operator $\hat{S}_\mathrm{l}$ which describes the behaviour of the field oscillators under interaction with the three-level medium. The scheme of constructing the similar operator can be found, for example, in work [5], where all the required details are given and here we repeat only basic points for construction.

Let the density matrix $\hat{F}$ describe the behaviour of two coherent quantum fields and a single three-level atom. Then we can write some obvious equation in the form:

$$\dot{\hat{F}} = -\imath[\hat{V}, \hat{F}] + \hat{R}_\mathrm{at}\hat{F}. \tag{36}$$

Here the interaction Hamiltonian

$$\hat{V} = \imath g_\mathrm{d} a_\mathrm{d}^m |a\rangle\langle c| + \imath g_\mathrm{l} a_\mathrm{l}|a\rangle\langle b| + \text{ h.c.}, \qquad m = 1, 2, 3, \ldots \tag{37}$$

looks like that of the semi-classical theory (2), but it is determined by the photon operators $a_\mathrm{l}, a_\mathrm{l}^\dagger$ and $a_\mathrm{d}, a_\mathrm{d}^\dagger$ instead of the respective complex amplitudes.

The operator $\hat{R}_\mathrm{at}$ ensures the incoherent processes $(a) \to (b)$ and $(b) \to (c)$ (in this section we use the model of the atom represented in the figure 1 without the additional level (d)).

In the Glauber diagonal representation, equation (36) is rewritten in the following form. In the Hamiltonian (37) the photon operators a_i and $a_i^\dagger$ are exchanged into their eigen-numbers α_i and α_i^* and to the right of the equation the additional terms arise. These terms are proportional to derivatives over the complex amplitudes α_d and α_l and they provide us with the possibility of constructing the iteration connected with their small magnitudes.

Next we factorize the density matrix $\hat{F}$ in the form:

$$\hat{F} = \rho\hat{\sigma} + \hat{\pi} \tag{38}$$

where the matrix $\hat{\sigma}$ is the atomic one describing the behaviour of our three-level atom in two 'classical' fields (in the Λ-configuration) with amplitudes α_d and α_l and this matrix obeys equation (2). The matrix

$$\rho = \mathrm{Tr}_\mathrm{atom}\,\hat{F} \tag{39}$$

is the field matrix for which we are trying to construct the kinetic equation, and $\hat{\pi}$ is the correlation matrix with property $\mathrm{Tr}\,\hat{\pi} = 0$. The factorization allows us to write the system of three equations for the matrices $\hat{\sigma}$, $\hat{\pi}$ and ρ instead of the single initial equation (36).

As is known, we have the possibility of constructing the kinetic equation for the field sub-system provided the atomic sub-system is developed much faster. This means that in the equations for ρ and $\hat{\pi}$, which are dependent on the matrix $\hat{\sigma}$, we may choose for the matrix $\hat{\sigma}$ the stationary solution. Simultaneously understanding that the matrix $\hat{\pi}$ that follows adiabatically to the ρ, we can substitute the stationary solution for $\hat{\pi}$ into the equation for ρ too. But even the stationary solution for $\hat{\pi}$ can be found only as a power series in derivatives over the complex amplitudes α_d and α_l. This means that in the general case the field kinetic equation contains in principle all the degrees of the derivatives. Nevertheless the obtained expressions give the possibility of calculating a development of the laser field state at the expense of a single atom. To take into account a lot of atoms we need simply to multiply the right-hand side of equation by the number of actual atoms N.

3.2. *Approximation of small photon fluctuations*

To make a mathematical situation simpler we can apply the approximation of small photon fluctuations in each mode. This means that we put

$$u_i = n_i + \varepsilon_i, \qquad n_i \gg \varepsilon_i \tag{40}$$

where n_i are stationary number of photons in i-mode inside the cavity.

Then selecting the photon matrix

$$R(\varepsilon_\mathrm{d}, \varepsilon_\mathrm{l}, t) = \int \mathrm{d}\varphi_\mathrm{d}\,\mathrm{d}\varphi_\mathrm{l}\,\rho(\varepsilon_\mathrm{d}, \varepsilon_\mathrm{l}, \varphi_\mathrm{l}, \varphi_\mathrm{d} t) \tag{41}$$

the equation for this matrix reads:

$$\frac{\partial}{\partial t}R(\varepsilon_\mathrm{d}, \varepsilon_\mathrm{l}, t) = \Gamma_\mathrm{d}\frac{\partial}{\partial\varepsilon_\mathrm{d}}(\varepsilon_\mathrm{d} - \delta_\mathrm{dl}\varepsilon_\mathrm{l})R + \Gamma_\mathrm{d} n_\mathrm{d}\xi_\mathrm{d}\frac{\partial^2 R}{\partial\varepsilon_\mathrm{d}^2} + \Gamma_\mathrm{l}\frac{\partial}{\partial\varepsilon_\mathrm{l}}(\varepsilon_\mathrm{l} - \delta_\mathrm{ld}\varepsilon_\mathrm{d})R - \frac{1}{2}\Gamma_\mathrm{l} n_\mathrm{l}\frac{\partial^2 R}{\partial\varepsilon_\mathrm{l}^2} + D\frac{\partial^2 R}{\partial\varepsilon_\mathrm{d}\,\partial\varepsilon_\mathrm{l}} + \{\cdots\}. \tag{42}$$

The coefficients are expressed via physical parameters in the form:

$$\Gamma_\mathrm{d} = \Gamma_0 + m\bar{\kappa}_\mathrm{d} \tag{43}$$

$$\delta_\mathrm{dl}\Gamma_\mathrm{d} = m\kappa_\mathrm{l} \tag{44}$$

$$\Gamma_\mathrm{l} = 2\kappa_\mathrm{l} \tag{45}$$

$$\delta_\mathrm{ld}\Gamma_\mathrm{l} = \bar{\kappa}_\mathrm{d} \tag{46}$$

$$\xi_\mathrm{d} = \frac{\xi_0(\Gamma_0 + \bar{\kappa}_\mathrm{d}) - (m-1)\bar{\kappa}_\mathrm{d}/2}{\Gamma_0 + m\bar{\kappa}_\mathrm{d}} \tag{47}$$

$$D = m\kappa_\mathrm{l} n_\mathrm{l}. \tag{48}$$

Here the parameters Γ_0 and ξ_0 connect with the drive laser: Γ_0 is the rate of damping the photon fluctuations there and ξ_0 is the respective statistical Mandel parameter. If we choose a sub-Poissonian laser investigated in [2] these coefficients read:

$$\Gamma_0 = \kappa_\mathrm{d}\frac{I_0}{1 + I_0}, \qquad \xi_0 = \frac{1}{I_0} - \frac{1}{2} \tag{49}$$

where I_0 is a dimensionless intensity of the drive and κ_d is the spectral width of mode. To choose a Poissonian laser we need to put $\xi_0 = 1/I_0$. So in saturation $I_0 \gg 1$, $\Gamma_0 = \kappa_\mathrm{d}$ and $\xi_0 = 0$ for Poissonian and $\xi_0 = -1/2$ for sub-Poissonian lasers.

In equation (42) the symbol $\{\cdots\}$ represents all the higher degrees of derivatives relative to fluctuations ε_l and ε_d. We need to keep them, in principle, for non-classical fields. At the same time we will demonstrate later that they do not contribute to our output signal given by formula (50) in the next section.

4. Photodetecting the radiation of the three-level laser

As is known, the photocurrent spectrum $i_\omega^{(2)}$ under detection, a single mode field in the approximation of small photon fluctuations, is given by the formula [6]:

$$i_\omega^{(2)}/i_{\rm shot}^{(2)} = 1 + \frac{\kappa_1}{n_1} 2\,{\rm Re} \int_0^\infty \langle \varepsilon_1(0)\varepsilon_1(t)\rangle {\rm e}^{\imath\omega t}\,{\rm d}t. \tag{50}$$

The first term on the right is equal to one, and represents the so-called Shotki current in the photodetection or the shot noise, the second one is traditionally called excess noise; nevertheless it is possible for it to be negative for quantum fields. One can see that to look for the photocurrent spectrum we need to calculate the correlation function $\langle \varepsilon_1(0)\varepsilon_1(t)\rangle$. For that it is enough to have the master equation in the form (42). On the basis of standard approaches from here, the system of the differential equations follows:

$$\frac{\rm d}{{\rm d}t}\langle \varepsilon_1(0)\varepsilon_1(t)\rangle = -2\kappa_1 \langle \varepsilon_1(0)\varepsilon_1(t)\rangle + \tilde{\kappa}_{\rm d}\langle \varepsilon_1(0)\varepsilon_{\rm d}(t)\rangle \tag{51}$$

$$\frac{\rm d}{{\rm d}t}\langle \varepsilon_1(0)\varepsilon_{\rm d}(t)\rangle = -(\Gamma_0 + m\tilde{\kappa}_{\rm d})\langle \varepsilon_1(0)\varepsilon_{\rm d}(t)\rangle + m\kappa_1\langle \varepsilon_1(0)\varepsilon_1(t)\rangle. \tag{52}$$

It is clear that the solutions of these equations depend on the initial conditions, that is, on the values $\langle \varepsilon_1^2(0)\rangle$, $\langle \varepsilon_{\rm d}^2(0)\rangle$ and $\langle \varepsilon_1(0)\varepsilon_{\rm d}(0)\rangle$. Because we mean the stationary light flux these values turn out to be independent of time and can be calculated for $t \to \infty$.

Then, on the basis of the same equation (42) we can obtain:

$$\frac{\rm d}{{\rm d}t}\langle \varepsilon_1^2\rangle = -4\kappa_1\langle \varepsilon_1^2\rangle + 2\tilde{\kappa}_{\rm d}\langle \varepsilon_1\varepsilon_{\rm d}\rangle - 2\kappa_1 n_1 = 0 \tag{53}$$

$$\frac{\rm d}{{\rm d}t}\langle \varepsilon_1\varepsilon_{\rm d}\rangle = -(\Gamma_0 + m\tilde{\kappa}_{\rm d} + 2\kappa_1)\langle \varepsilon_1\varepsilon_{\rm d}\rangle + m\kappa_1\langle \varepsilon_1^2\rangle + \tilde{\kappa}_{\rm d}\langle \varepsilon_{\rm d}^2\rangle + m\kappa_1 n_1 = 0 \tag{54}$$

$$\frac{\rm d}{{\rm d}t}\langle \varepsilon_{\rm d}^2\rangle = -2(\Gamma_0 + m\tilde{\kappa}_{\rm d})\langle \varepsilon_{\rm d}^2\rangle + 2m\kappa_1\langle \varepsilon_1\varepsilon_{\rm d}\rangle - 2(\Gamma_0 + m\tilde{\kappa}_{\rm d})n_{\rm d}\xi_{\rm d} = 0. \tag{55}$$

Here it needs to be stressed that the systems (53)–(55) and (51), (52) turn out to be perfectly independent of the terms of the basic equation (42), represented by the symbol $\{\cdots\}$, although they were written without any restrictions relative to them. They simply do not contribute to these systems in the exact sense of word.

Relatively simple calculations allow us to write the explicit expression for a photocurrent spectrum in the general case:

$$i_\omega^{(2)}/i_{\rm shot}^{(2)} = 1 - 2\kappa_1^2 \times \frac{\omega^2 + \Gamma_0(\Gamma_0 + m\tilde{\kappa}_{\rm d}) - m\tilde{\kappa}_{\rm d}(\Gamma_0 + \tilde{\kappa}_{\rm d})\xi_0 + m(m-1)\tilde{\kappa}_{\rm d}^2/2}{[\kappa_1(2\Gamma_0 + m\tilde{\kappa}_{\rm d}) - \omega^2]^2 + \omega^2(\Gamma_0 + m\tilde{\kappa}_{\rm d} + 2\kappa_1)^2}. \tag{56}$$

For our level of interest it is enough to discuss two particular cases. We will speak about the weak connection between the driving and driven lasers, if the rate of escape of photons out of the cavity of the drive laser, and the rate of damping the photon fluctuations there, are independent of the presence of the driven three-level laser. It is achieved provided

$$\Gamma_0 \gg m\tilde{\kappa}_{\rm d}(\geqslant \kappa_{\rm d}). \tag{57}$$

By contrast the case of the strong connection, when the rates are determined solely by absorption in the three-level medium, is realized under the condition

$$\Gamma_0 \ll \tilde{\kappa}_{\rm d}(\leqslant m\kappa_{\rm d}). \tag{58}$$

It is not difficult to see that the photocurrent spectrum for the weak connection is given by the formula:

$$i_\omega^{(2)}/i_{\rm shot}^{(2)} = \frac{\omega^4 + \omega^2(\Gamma_0^2 + 2\kappa_1^2) + 2\kappa_1^2\Gamma_0^2}{\omega^4 + \omega^2(\Gamma_0^2 + 4\kappa_1^2) + 4\kappa_1^2\Gamma_0^2}. \tag{59}$$

Apparently, under the high frequencies, the level of noise is shot because $i_\omega^{(2)}/i_{\rm shot}^{(2)} \to 1$. At the same time, at zero frequency $i_\omega^{(2)}/i_{\rm shot}^{(2)} \to 1/2$. This means that sub-Poissonian lasing takes place with reduction of the shot noise by half. Moreover this fact does not depend on the photon statistics of the drive laser (there is no dependence on ξ_0) and we have the same effect as there was before [1].

In the case of the strong connection the photocurrent spectrum reads:

$$i_\omega^{(2)}/i_{\rm shot}^{(2)} = 1 - \kappa_1^2 \frac{2\omega^2 - m\tilde{\kappa}_{\rm d}^2(1 + 2\xi_0 - m)}{(\omega^2 + m\tilde{\kappa}_{\rm d}\kappa_1)^2 + \omega^2(m^2\tilde{\kappa}_{\rm d}^2 + 4\kappa_1^2)}. \tag{60}$$

As is seen as for the weak connection, the shot noise takes place for high enough frequencies. At the same time the reduction at zero frequency is given by the formula:

$$i_{\omega=0}^{(2)}/i_{\rm shot}^{(2)} = \frac{2\xi_0 + 1}{m} \tag{61}$$

and now depends on the photon statistics of the drive laser and on the number m. If at first we choose $m = 1$ we can conclude that in the case of strong connection between lasers the reduction of shot noise at zero frequency is the same as for the drive laser. In fact we have the perfect reduction with $\xi_0 = -1/2$ and the zero one for the Poissonian drive $\xi_0 = 0$. At the same time we can achieve appreciable reduction even for the Poissonian drive under $m \gg 1$.[4] This effect can be understood on the basis of the following simple consideration. We could discuss our three-level system as a system which converts the m photons in the channel of driving into one photon in the laser channel. The simplest calculation demonstrates that under this operation the Poissonian flux is converted into the sub-Poissonian one.

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[4] This result was discussed for the first time in [7].

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